

# Phase structure of U(1) lattice gauge theory with monopole term

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## Abstract

We investigate four-dimensional compact U(1) lattice gauge theory with a monopole term added to the Wilson action. First we consider the phase structure at negative  $\beta$ , revealing some properties of a third phase region there, in particular the existence of a number of different states. Then our present studies concentrate on larger values of the monopole coupling  $\lambda$  where the confinement-Coulomb phase transition turns out to become of second order. Performing a finite-size analysis we find that the critical exponent  $\nu$  is close to, however, different from the gaussian value and that in the range considered  $\nu$  increases somewhat with  $\lambda$ .

# I. INTRODUCTION

The phase transition in 4-dimensional compact U(1) lattice gauge theory with the Wilson action because of the occurrence of a gap in the energy histogram is believed to be of first order. Actually this is to be studied in more detail by a finite-size analysis. Recently critical exponents have been determined in a higher-statistics study [1]. There the critical exponent  $\nu$  has been found to decrease towards  $\frac{1}{4}$  with increasing lattice size, i.e. towards the value characteristic of a first-order transition.

If the Wilson action is extended by a double charge term with coupling  $\gamma$  the first order transition weakens with decreasing  $\gamma$  and has been conjectured to become of second order at some negative  $\gamma$  [2], which, however, so far has not been confirmed. In Ref. [3] instead of the usual periodic boundary conditions a spherelike lattice (i.e. the surface of a 5-cube which is homeomorphic to a 4-sphere) has been used. The disappearance of the gap at  $\gamma = 0, -0.2, -0.5$  observed there has been interpreted by the authors as an earlier start of the second order region and the relatively low value of  $\nu$  there as related to a nongaussian fixed point.

In a more recent investigation of the action with double charge term [4] the gap has been shown for  $\gamma$  down to  $-0.4$  to reappear on larger spherelike lattices. The absence of a gap on smaller lattices has been attributed to larger finite-size effects in the spherical geometry. These observations are just what was to be expected according to studies of the influence of different boundary conditions [5] where it turned out that inhomogeneities weaken the transition and that the gap reappears for sufficiently large lattice sizes. In addition in Ref. [4] for  $\gamma$  between  $+0.2$  and  $-0.4$  the critical exponent  $\nu$  has been found to decrease towards  $\frac{1}{4}$  with increasing lattice size for toroidal as well as for spherical geometry. Further, in some cases stabilization of the latent heat has been observed.

Thus, for the above actions there are now rather strong indications that, at least in the region which has been investigated so far, the transition is of first order. Of course, it remains desirable to check this on still larger lattices. While this is hardly possible with conventional methods, it can be done using the dynamical-parameter algorithm developed in [6, 7].

After the mentioned evidence of first order the question arises where one can find a second-order phase transition in the U(1) gauge system. This leads to the modification of the Wilson action where a monopole term is added [8], which reads

$$S = \beta \sum_{\mu > \nu, x} (1 - \cos \Theta_{\mu\nu, x}) + \lambda \sum_{\rho, x} |M_{\rho, x}| \quad (1.1)$$

with  $M_{\rho,x} = \epsilon_{\rho\sigma\mu\nu}(\bar{\Theta}_{\mu\nu,x+\sigma} - \bar{\Theta}_{\mu\nu,x})/4\pi$  where the physical flux  $\bar{\Theta}_{\mu\nu,x} \in [-\pi, \pi)$  is related to the plaquette angle  $\Theta_{\mu\nu,x} \in (-4\pi, 4\pi)$  by  $\Theta_{\mu\nu,x} = \bar{\Theta}_{\mu\nu,x} + 2\pi n_{\mu\nu,x}$  [9].

Using the action (1.1) from the energy distribution it is seen that the gap gets smaller with increasing monopole coupling  $\lambda$  [6, 7], which indicates that the first-order transition gets weaker and for sufficiently large  $\lambda$  becomes of second order. The latter has been corroborated by a finite-size analysis [10] which has shown that at  $\lambda = 0.9$  the critical exponent is already characteristic of a second-order transition.

In addition to showing a second-order phase transition for sufficiently large  $\lambda$ , the action (1.1) is particularly attractive in view of the fact [6, 11] that the confinement phase and the Coulomb phase are unambiguously characterized by the presence or absence, respectively, of an infinite network of monopole current lines, where “infinite” on finite lattices is to be defined in accordance with the boundary conditions [5]. Since the probability  $P_{\text{net}}$  to find an infinite network takes the values 1 and 0, respectively, it is very efficient to discriminate between those phases. For the finite lattices with periodic boundary conditions we are considering here, “infinite” means “topologically nontrivial in all directions”. While for loops this characterization is straightforward, to determine the topological properties of the occurring networks it is necessary to perform a more elaborate analysis based on homotopy preserving mappings [6, 11].

In [10] the location  $\beta_{\text{cr}}$  of the transition from the confinement phase to the Coulomb phase has been determined as a function of  $\lambda$ . It has been found that the related transition line continues to negative  $\beta$ . On the other hand, in Refs. [12, 13] a further transition has been reported at  $\beta = -1$  for  $\lambda = 0$  and at  $\beta = -0.7$  for  $\lambda = \infty$ . This suggests to check in more detail what happens at negative  $\beta$ .

In the second-order region mentioned above an important question is whether one has universal critical properties there. The energy distribution indicates that this region starts at some finite  $\lambda$  above 0.7 [6, 7]. For  $\lambda = 0.9$  the critical exponent  $\nu$  is known to be characteristic of a second-order transition [10]. It appears important now to get more information on this region. In particular finite-size analyses at different values of  $\lambda$  and for a variety of observables are desirable.

In the present work we have performed Monte Carlo simulations to address the indicated questions. Simulation runs at various values of  $\lambda$  and  $\beta$  have been done to get an overview of the situation at negative  $\beta$ . The emerging picture of this is discussed in Sect. II. The emphasis of our investigations has then been on higher-statistics simulations at  $\lambda = 1.1$  and  $\lambda = 0.8$  in the critical region of the confinement-Coulomb phase transition, which have been evaluated by finite-size analyses. The results of this are presented in Sect. III.

## II. PHASE REGIONS

In [10] the location  $\beta_{\text{cr}}$  of the transition from the confinement phase to the Coulomb phase has been determined by the maximum of the specific heat  $C_{\text{max}}$  up to  $\lambda = 1.3$ . Because the peak of the specific heat has been found to decrease strongly with  $\lambda$ , at larger values of  $\lambda$  only  $P_{\text{net}}$  has been used to locate the transition. In these investigations it has turned out that above  $\lambda = 1.2$  the associated values of  $\beta_{\text{cr}}$  become negative.

The Wilson action has the symmetry  $\beta \rightarrow -\beta$ ,  $U_{\square} \rightarrow -U_{\square}$ <sup>1</sup>. At  $\lambda = 0$  it gives rise to a phase transition at  $\beta = -1$  in addition to the one at  $\beta = 1$  [12]. For  $\lambda \neq 0$  this symmetry is violated by the monopole term. At  $\lambda = \infty$  only the transition at negative  $\beta$  persists and occurs at about  $\beta = -0.7$  [12, 13].

Here we have checked the occurrence of such transition at negative  $\beta$  also at intermediate values of  $\lambda$  by determining  $C_{\text{max}}$ . Our observations indicate a transition line extending from  $(\lambda, \beta) = (0, -1)$  to  $(\infty, -0.7)$ . In the energy distribution on the  $8^4$  lattice we have seen a double-peak structure at  $\lambda = 0.5$  and indications of such structures (though not resolved with present accuracy) at other  $\lambda$  values. Together with the observations at  $\lambda = 0$  and at  $\lambda = \infty$  in Refs. [12, 13] this hints at first order along that transition line. Of course, it remains to confirm this with higher statistics and on larger lattices.

We have also investigated some properties of the system below the new transition line which will be discussed in the following. Since so far we have no indication of a further subdivision of this region, but rather find similar properties throughout it, we consider it here (at least as a working hypothesis) as a third phase.

Figure 1 gives an overview of the phase regions as they are according to our present knowledge. The line separating confinement and Coulomb phases obtained in [10] with  $C_{\text{max}}$  has been supplemented by a point at  $\lambda = 1.35$  determined here and by the point  $(1.4, 0.52)$  found there only using  $P_{\text{net}}$ . It is seen that this transition line hits the boundary of the third phase at approximately  $(1.43, -0.71)$ .

While  $P_{\text{net}}$  has provided an unambiguous criterion in the confinement phase and in the Coulomb phase, this is no longer so in the third phase. At fixed  $(\lambda, \beta)$  in this region values 0 as well as 1 occur for  $P_{\text{net}}$ . The observations at  $(10, -1000)$  in [10] appear to be related to this. That properties of monopole structures are no

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<sup>1</sup>The transformation  $U_{\square} \rightarrow -U_{\square}$  of the plaquette fields can be realized by transforming the link angles as  $\Theta_{\mu x} \rightarrow \pm \Delta_{\mu x} \Theta_{\mu x}$  where the sign choice is according to  $\Theta_{\mu x}$  being in the intervals  $[-\pi, 0)$  and  $[0, \pi)$ , respectively, and where  $\Delta_{\mu x} = -1$  if  $x_1 + x_2 + \dots + x_{\mu-1}$  is odd and  $\Delta_{\mu x} = +1$  if it is even or if  $\mu = 1$ .

longer characteristic at sufficiently negative  $\beta$  can be understood by noting that the respective quantities are not invariant under the transformation  $\beta \rightarrow -\beta$ ,  $U_\square \rightarrow -U_\square$ .

As a characteristic feature of the third phase we have found that a number of different states exists there between which transitions in the simulations are strongly suppressed. We have observed this phenomenon at various negative  $\beta$  in the  $\lambda$  range from 0 to 2.5. Typical examples of time histories of the average plaquette  $\epsilon = (1/6L^4) \sum_{\mu>\nu,x} (1 - \cos \Theta_{\mu\nu,x})$  are given in Figure 2, with the results of seven simulation runs, exhibiting different states and some transitions from lower to higher ones. It turns out that there is no correlation between the state reached and the type of start (hot or cold) of a simulation run.

Comparing time histories of  $\epsilon$  at different  $(\lambda, \beta)$  the splittings of the states appear very similar. Closer inspection shows that at least nine states occur. The number of transitions between states increases with  $\lambda$ . They have so far always been observed occurring from lower to higher plaquette values. The widths in the time histories show little dependence on  $\lambda$  while they strongly decrease if  $\beta$  gets more negative (width  $\times \beta$  being roughly constant). Thus at smaller negative  $\beta$  it gets increasingly difficult to resolve splittings.

The average  $\epsilon$  found show not much dependence on  $\lambda$ , however, they increase as  $\beta$  gets more negative. The  $\epsilon$  values, being somewhat below 2, indicate that at sufficiently negative  $\beta$  the average of  $\cos \Theta_{\mu\nu,x}$  gets close to  $-1$ . That negative  $\cos \Theta_{\mu\nu,x}$  occur at negative  $\beta$  and positive  $\cos \Theta_{\mu\nu,x}$  at positive  $\beta$  in view of the symmetry  $\beta \rightarrow -\beta$ ,  $U_\square \rightarrow -U_\square$  of the Wilson action is conceivable.

Though at negative  $\beta$  the monopole density  $\rho = (1/4L^4) \sum_{\rho,x} |M_{\rho,x}|$  appears of less interest, it should be mentioned that the states are also seen in its time histories. The respective time histories are correlated with large values of  $\rho$  corresponding to low values of  $\epsilon$ . The widths for  $\rho$  decrease with  $\lambda$ . They are rather large so that, except at the largest values of  $\lambda$ , the resolution of the states is very poor. The value of  $\rho$  decreases with  $\lambda$ .

The origin of the observed different states is not yet clear. Similar observations are made in simulations of spin glasses and of frustrated systems, and also in finite-temperature SU(N) gauge theory with states related to spontaneous  $Z(N)$  breaking. In any case the third phase appears to have more complicated properties which deserve further (though computationally demanding) studies elsewhere.

### III. CRITICAL PROPERTIES

Monte Carlo simulations with about  $10^6$  sweeps have been performed at each of up to 20 values of  $\beta$  in the critical region of the confinement-Coulomb phase transition at  $\lambda = 1.1$  and at  $\lambda = 0.8$  and for each lattice size considered. Multihistogram techniques [14] have been applied in the evaluation of the data. The errors have been estimated by Jackknife methods. In the finite-size analyses in addition to the specific heat and the Challa-Landau-Binder (CLB) cumulant [15] complex zeros of the partition function [16], in particular the Fisher zero  $z_0$  closest to the  $\beta$  axis [17], have been used.

The specific heat is

$$C = \frac{1}{6L^4} \langle (E - \langle E \rangle)^2 \rangle \quad (3.1)$$

where  $E = \sum_{\mu > \nu, x} (1 - \cos \Theta_{\mu\nu, x})$ . For  $d = 4$  its maximum is expected to behave as

$$C_{\max} \sim L^4 \quad (3.2)$$

if the phase transition is of first order and as

$$C_{\max} \sim L^{\frac{\alpha}{\nu}} \quad (3.3)$$

if it is of second order, where  $\alpha$  is the critical exponent of the specific heat and  $\nu$  the critical exponent of the correlation length.

In Figure 3 we present the results for  $C_{\max}$  obtained on lattices with  $L = 6, 8, 10, 12, 16$  for  $\lambda = 0.8, 0.9, 1.1$ . They include data from simulations of the present investigation ( $\lambda = 1.1$  and  $\lambda = 0.8$ ) and ones from simulations of Ref. [10] ( $\lambda = 0.9$ ). From Figure 3 it is already obvious that  $\frac{\alpha}{\nu}$  as it occurs in (3.3) decreases with  $\lambda$ , which by the hyperscaling relation  $\alpha = 2 - d\nu$  means that  $\nu$  increases with  $\lambda$ .

The fits to the data presented in Figure 3 give the values for  $\frac{\alpha}{\nu}$  shown in Table I. They are clearly very far from 4 and thus the transition cannot be expected to be of first order. Using the hyperscaling relation  $\alpha = 2 - d\nu$  the values for  $\nu$  listed in Table II are obtained. They are different from the value  $\frac{1}{2}$  of the gaussian case, however, rather close to it. Thus in any case to conclude on second order appears quite safe.

Similar results are obtained for the minimum of the CLB cumulant

$$U_{\text{CLB}} = \frac{1}{3} \left( 1 - \frac{\langle E^4 \rangle}{\langle E^2 \rangle^2} \right) \quad (3.4)$$

and for the imaginary part of the closest Fisher zero  $z_0$ . For these quantities finite-size scaling predicts the behaviors

$$\text{Im}(z_0) \sim L^{-\frac{1}{\nu}} , \quad (3.5)$$

$$U_{\text{CLB},\min} \sim L^{\frac{\alpha}{\nu}-4} . \quad (3.6)$$

The results of the respective fits are also listed in Tables I and II. It is seen that the values obtained from different quantities roughly agree, with some systematic deviations beyond the given statistical errors. In any case it is obvious that  $\nu$  increases somewhat with  $\lambda$  and that it is not far from  $\frac{1}{2}$ .

The related critical  $\beta_{\text{cr}}(L)$  (i.e. the extrema positions of  $C$  and  $U_{\text{CLB}}$  and the real part of  $z_0$ ) are given in Table III. They illustrate the dependence on  $\lambda$  and lattice size. The critical  $\beta_{\text{cr}}(L)$  are expected to behave as

$$\beta_{\text{cr}}(L) = \beta_{\text{cr}}^{\infty} + aL^{-\frac{1}{\nu}} . \quad (3.7)$$

Using the values of  $\nu$  in Table II and the data for  $\beta_{\text{cr}}(L)$  in Table III, the numbers  $\beta_{\text{cr}}(\infty)$  and  $a$  in (3.7) have been calculated for  $\lambda=1.1$ . In Table IV they are compared with those obtained for  $\lambda=0.9$  in [10]. The errors given are again only statistical ones.

The observed increase of  $\nu$  with  $\lambda$  could indicate a nonuniversal behavior with a maximal value (which could be even  $\frac{1}{2}$ ) reached at the boundary to the third phase. Another possibility is that, because the observations are on finite lattices and the range of  $\lambda$  considered is not too far from the first-order region, the starting point of the second-order region is not yet sharp. Then this effect should disappear on much larger lattices.

To get hints on the dependence on lattice size effective  $\nu$  (i.e. a sequence of ones related to two neighboring lattice sizes) have also been considered. Though there was a slight tendency in support of the above view of a finite-size effect, it turns out that larger lattices and higher statistics (which unfortunately are computationally very expensive) would be needed for firm conclusions.

If for very large lattices  $\nu$  gets independent of  $\lambda$  for the  $\lambda$  above a certain finite value and below the start of the third phase, the question arises what the ultimate  $\nu$  would be. Because the numbers for  $\nu$  in Table II come rather close to  $\frac{1}{2}$  this could be simply the gaussian value. If first order is confirmed along the boundary to the third phase, an interesting aspect is that this transition line with  $\nu = \frac{1}{4}$  is met by the second-order line with  $\nu$  close to or equal to  $\frac{1}{2}$ .

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**TABLE I**

Critical exponents  $\frac{\alpha}{\nu}$  from  $C_V$ ,  $U_{\text{CLB}}$ .

$\lambda$	$C_V$	$U_{\text{CLB}}$
0.8	0.616(22)	0.756(23)
0.9	0.485(35)	
1.1	0.284(12)	0.391(12)

**TABLE II**

Critical exponents  $\nu$  from  $\text{Im}(z_0)$ ,  $C_V$ ,  $U_{\text{CLB}}$ .

$\lambda$	$\text{Im}(z_0)$	$C_V$	$U_{\text{CLB}}$
0.8	0.404(5)	0.433(2)	0.421(3)
0.9		0.446(5)	
1.1	0.421(8)	0.467(2)	0.455(2)

**TABLE III**Critical  $\beta_{\text{cr}}$  from  $z_0$ ,  $C_V$ ,  $U_{\text{CLB}}$ .

$\lambda$	$L$	$\text{Re}(z_0)$	$C_V$	$U_{\text{BCL}}$
0.8	6	0.4637 (3)	0.4630 (3)	0.4647 (2)
	8	0.4783 (2)	0.4779 (2)	0.4783 (4)
	10	0.4844 (2)	0.4841 (2)	0.4843 (2)
1.1	6	0.1409 (10)	0.1387(5)	0.1431 (6)
	8	0.1636 (10)	0.1618(5)	0.1630 (6)
	10	0.1742 (4)	0.1731(5)	0.1736 (6)
	12	0.1775 (8)	0.1782(8)	0.1784 (8)
	16	0.1839 (4)	0.1837(4)	0.1838 (4)

**TABLE IV**Fit data  $\beta_{\text{cr}}^\infty$  and  $a$  for  $z_0$ ,  $C_V$ ,  $U_{\text{CLB}}$ .

$\lambda$		$\text{Re}(z_0)$	$C_V$	$U_{\text{CLB}}$
0.9	$\beta_{\text{cr}}^\infty$		0.4059(5)	
	$a$		-1.99(6)	
1.1	$\beta_{\text{cr}}^\infty$	0.1888 (6)	0.1902 (5)	0.1896 (5)
	$a$	-3.14(13)	-2.41 (7)	-2.49 (8)

## Figure captions

- Fig. 1. Location of phase transition points  $\beta_{\text{cr}}$  on  $8^4$  lattice as function of  $\lambda$  between confinement and Coulomb phases (circles from  $C_{\text{max}}$ , square from  $P_{\text{net}}$ ) and to third phase (diamonds). Curves drawn to guide the eye.
- Fig. 2. Typical time histories of the average plaquette  $\epsilon$  at negative  $\beta$  obtained for seven different simulation runs, shown for  $(\lambda, \beta) = (2, -20)$  and  $8^4$  lattice.
- Fig. 3. Maximum of specific heat  $C_{\text{max}}$  as function of lattice size  $L$  for  $\lambda = 0.8, 0.9$ , and  $1.1$  at transition points  $\beta_{\text{cr}}$  between confinement and Coulomb phases.

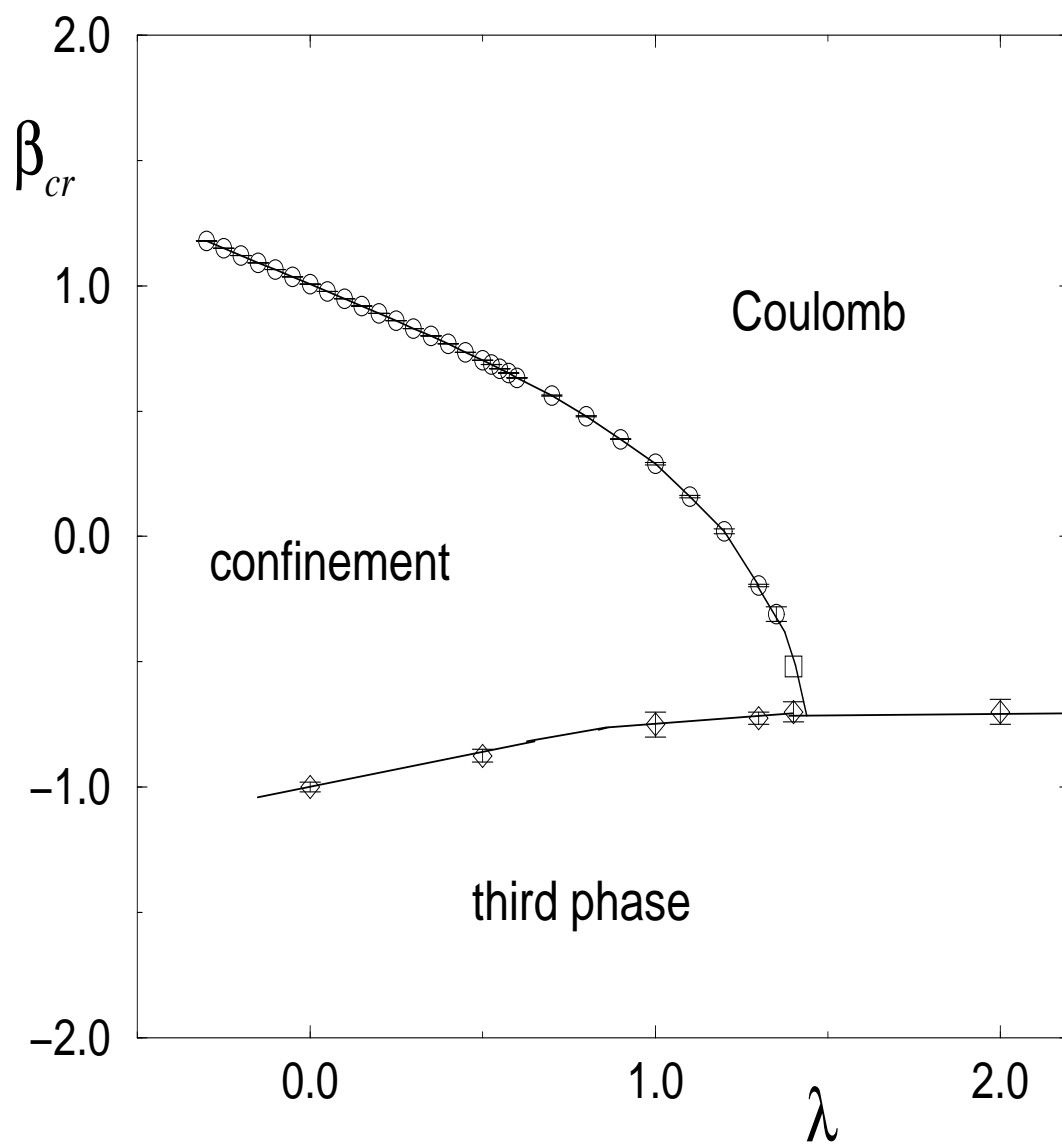


Figure 1:

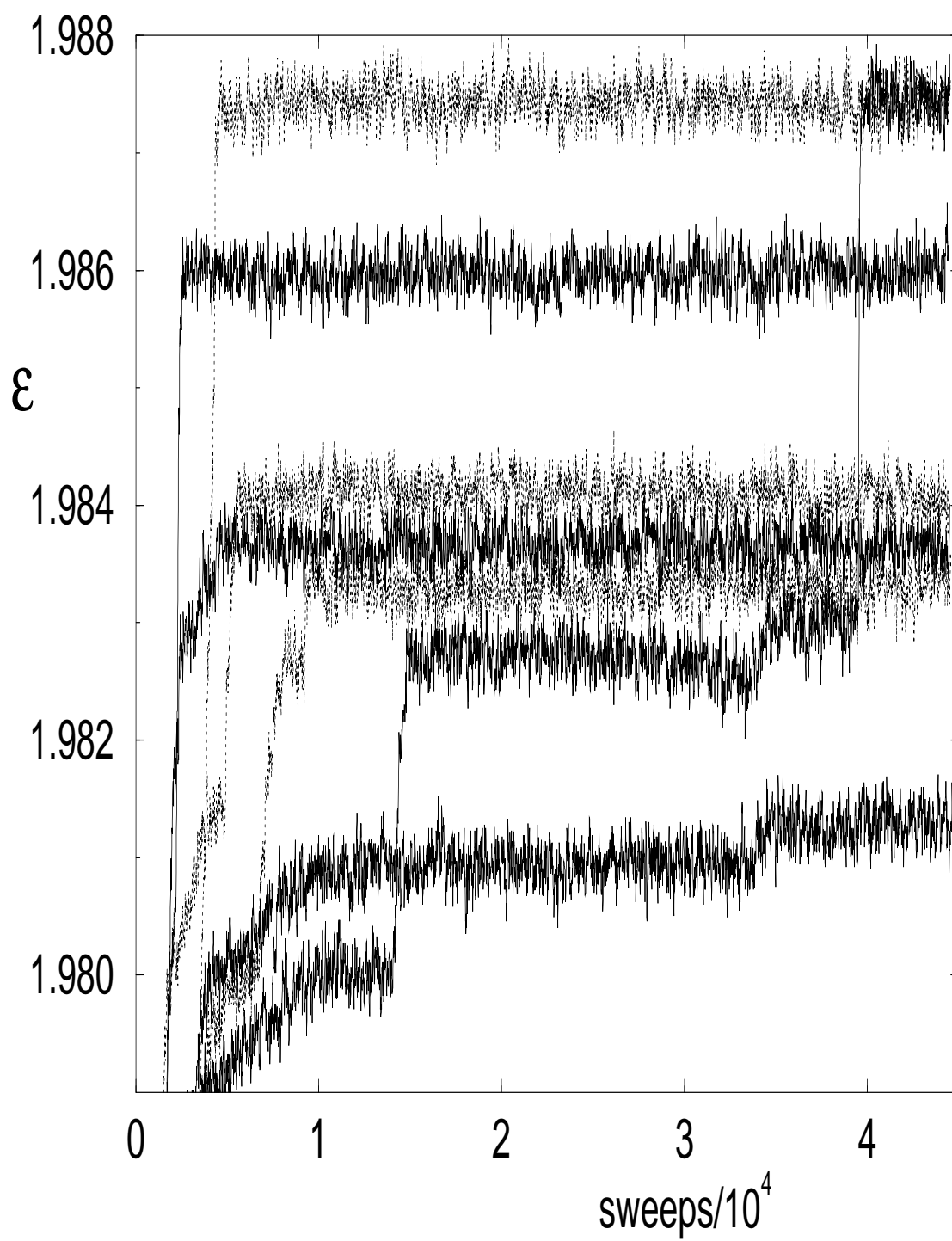


Figure 2:

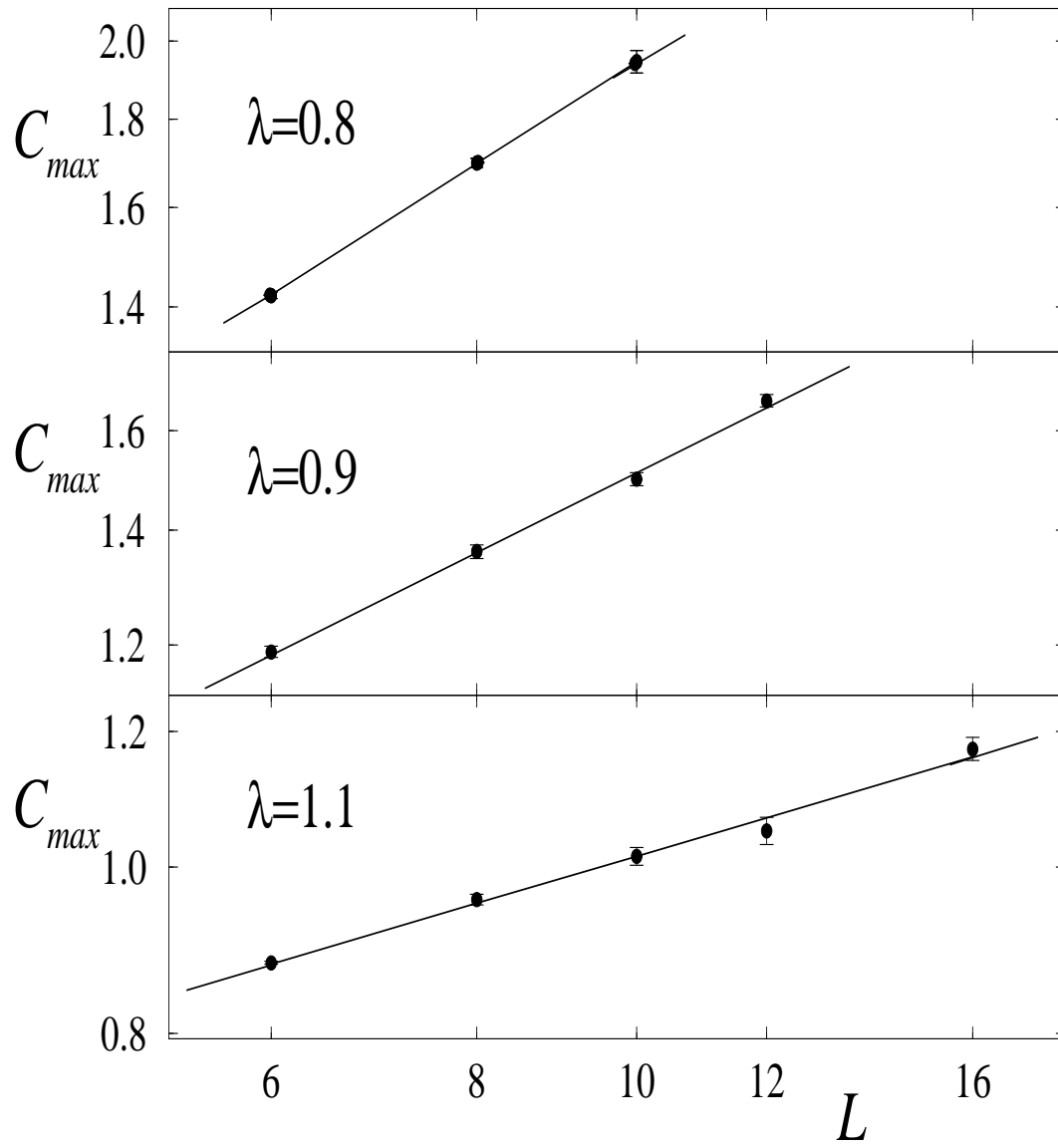


Figure 3: